# Reflection of light from semi-infinite turbid media

**Alexander A. Kokhanovsky**

*Institute of Particle Technology and Environmental Engineering, Technical University of Clausthal, Leibnizstrasse 19, D-38678 Clausthal-Zellerfeld, Germany, and Stepanov Institute of Physics, National Academy of Sciences of Belarus, 68 F. Scarina Avenue, 220072 Minsk, Belarus*

Received December 2, 1997; revised manuscript received July 10, 1998; accepted July 13, 1998

The simple equation for the spherical albedo of a semi-infinite turbid medium is obtained. The accuracy of the approximation was studied with the numerical solution of the radiative transfer equation. The error is smaller than 10% for water clouds in the visible and the near infrared. © 1998 Optical Society of America [S0740-3232(98)00111-2]

*OCIS codes:* 290.7050, 120.5700, 030.5620.

Studies of light reflection from semi-infinite turbid media are important for many applications, including medical imaging and cloud, snow, and foam optics. The spherical albedo of such media $R$ can be obtained by integration of the reflection function $r(\mu, \mu_0, \psi)$ (Ref. 1):

$$R = \frac{2}{\pi}\int_0^1 \mu \mathrm{d}\mu \int_0^1 \mu_0 \mathrm{d}\mu_0 \int_0^{2\pi} r(\mu,\ \mu_0,\ \psi)\mathrm{d}\psi, \tag{1}$$

where $\mu = \cos \nu$; $\mu_0 = \cos \nu_0$; $\nu$ and $\nu_0$ are the observation and the incidence angles, respectively; and $\psi$ is the azimuth. The function $r(\mu,\ \mu_0,\ \psi)$ is the solution of the radiative transfer equation.[1] This solution depends on the single scattering albedo $\omega$ and the phase function $p(\theta)$, where $\theta$ is the scattering angle. The value of $\omega$ represents the probability that a photon injected into a turbid medium will be scattered after interaction with a particle. The function $p(\theta)$ is the probability that a photon will scatter in the direction specified by the scattering angle $\theta$. According to the similarity principle,[1] turbid media that have different phase functions $p(\theta)$ but the same value of the asymmetry parameter

$$g = \frac{1}{2}\int_0^{\pi} p(\theta)\cos\ \theta\ \sin\ \theta d\theta \tag{2}$$

and the single scattering albedo $\omega$ have approximately the same radiative characteristics. Thus in the first coarse approximation the value of $R$ depends only on two parameters, namely, $\omega$ and $g$.

An approximate formula for the spherical albedo of semi-infinite turbid media was obtained by Perelman *et al.*[2] within the framework of the path integral formalism:

$$R = \exp\left[-\alpha\left(\frac{\ln \frac{1}{\omega}}{1-g}\right)\right]^{1/2}, \tag{3}$$

where, according to Perelman *et al.*,[2] the value of $\alpha$ is a function of $g$.

The purpose of this paper is to show that the value of $\alpha$ should be constant and to compare Eq. (3) with exact radiative transfer calculations.

The following exact asymptotical relation for the spherical albedo of a semi-infinite medium can be obtained from the radiative transfer theory as $\omega \to 1$ (Ref. 1):

$$R = 1 - 4\left[\frac{1-\omega}{3(1-g)}\right]^{1/2}. \tag{4}$$

This relation is valid for any phase functions and values of $g$. It is possible to find the value of $\alpha$ in Eq. (3) by comparing Eqs. (4) and (3) as $\omega \to 1$. Indeed, it follows from Eq. (3) as $\omega \to 1$ that

$$R = 1 - \alpha\left(\frac{1-\omega}{1-g}\right)^{1/2}, \tag{5}$$

where the asymptotical formula

$$\lim_{\omega\to 1}\left[\ln\left(\frac{1}{\omega}\right)\right] = 1 - \omega \tag{6}$$

is used. Thus one can see that the value of $\alpha$ does not depend on $g$ and that

$$\alpha = \frac{4}{\sqrt{3}}. \tag{7}$$

The value of $\alpha$ in the paper by Perelman *et al.*[2] was approximately 3.1 at $g = 0.908$ and approximately 2.9 at $g = 0.757$. This value is nearly 30% larger than it should be according to Eq. (7). It follows from Eqs. (3) and (7) that

$$R = \exp\left\{-4\left[\frac{\ln \frac{1}{\omega}}{3(1-g)}\right]^{1/2}\right\}. \tag{8}$$

One can see that

$$\lim_{\omega\to 0} R(\omega) = 0, \quad \lim_{\omega\to 1} R(\omega) = 1, \tag{9}$$

as expected. The dependence $R(\omega)$, calculated with Eq. (8) at $g = 0.85$, is represented in Fig. 1. It is interesting that at $\omega > 0.9$ ($R > 0.2$) a somewhat simpler formula for calculation of the spherical albedo can be applied [see Eq. (6) and Fig. 1]:

$$R = \exp\left\{-4\left[\frac{1-\omega}{3(1-g)}\right]^{1/2}\right\}. \tag{10}$$

Equation (10) was obtained by Rozenberg[3] for a special case of weakly absorbing media. A comparison of the results of the spherical-albedo calculation obtained with Eq. (8) and with a numerical solution of the radiative transfer equation[1] for semi-infinite water clouds with the gamma particle size distribution

$$f(a) = Ba^6 \exp(-1.5a), \tag{11}$$

where $B$ is the normalization constant $[\int_0^\infty f(a)\mathrm{d}a = 1]$, is presented in Fig. 2. One can see that the accuracy of the approximate Eq. (8) is high.

In conclusion, a simple approximation (8) was proposed to calculate the spherical albedo of turbid semi-infinite media. This approximation can be used to avoid complex numerical calculations of the total reflectance $R$ and absorption $A = 1 - R$ of radiation by semi-infinite thick turbid media.

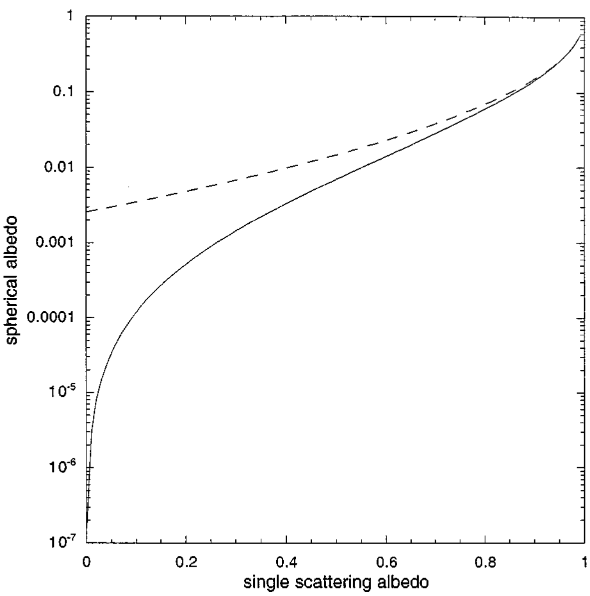


Fig. 1. Dependence of the spherical albedo of a semi-infinite medium on the single scattering albedo at $g = 0.85$, calculationed with Eq. (8) (solid curve) and with Eq. (10) (dashed curve).

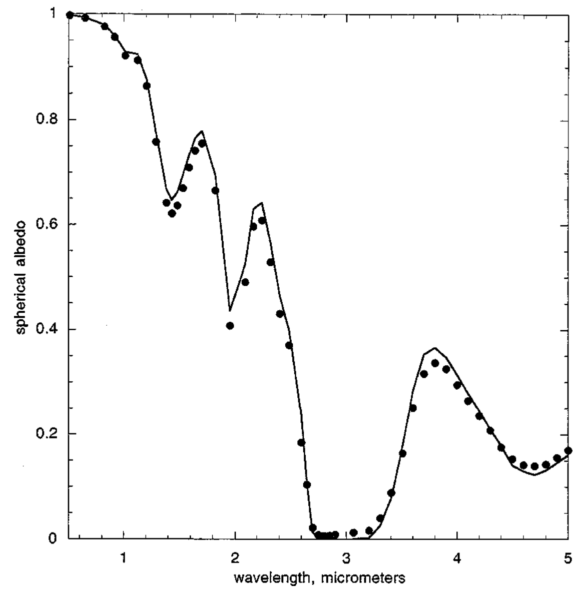


Fig. 2. Dependence of the spherical albedo of a semi-infinite water cloud with particle size distribution (11) on the wavelength, calculated with Eq. (8) (solid curve) and with the numerical solution of the radiative transfer equation (symbols).

## ACKNOWLEDGMENT

This research was supported by the Alexander von Humboldt Foundation.

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